Mathematics
UNIVERSITY OF TORONTO

2025-26 Graduate Course Proposals

If you wish to offer a **graduate course** for possible teaching credit for the 2025-2026 academic year, kindly use this form to enter your submission no later than **Friday, January 10, 2025**. You are encouraged to discuss with colleagues in your research area the courses you wish to offer as to coordinate the scheduling (fall vs. winter), the topics, and the level (introductory, intermediate or advanced).
Kindly refrain from entering your undergraduate course (including cross-listed) request in this form.
Many thanks.

A. Introduction

Welcome! This form has various sections; you can find which section is relevant to you by choosing an option in question 2 then skipping straight to the option that fits you best.

1. Name (Last, First) *

Bar-Natan, Dror

2. Course(s) that I would like to teach: Core, Fields Institute, Topics or Intermediate (**please go to relevant sections of the form**) *

- ○ Core course only (go to section 2)
- ○ Fields only (go to section 3)
- ○ Topics course only (go to section 4)
- ● Core and topics (go to sections 2 and 4)

- [ ] Intermediate Course only (go to section 5)
- [ ] All of the above

Core Courses

Please indicate your interest in teaching a core course in this section.

3. Core courses that I would like to teach:

- [ ] Real Analysis I (Fall semester)
- [ ] Partial Differential Equations I (Fall semester)
- [x] Algebra I (Fall semester)
- [x] Differential Topology (Fall semester)
- [ ] Mathematical Probability I (Fall semester)
- [ ] Linear Algebra and Optimization (Fall semester)
- [ ] Real Analysis II (Winter semester)
- [ ] Complex Analysis (Winter semester)
- [ ] Partial Differential Equations II (Winter semester)
- [ ] Algebra II (Winter semester)
- [x] Algebraic Topology (Winter semester)
- [ ] Mathematical Probability II (Winter semester)

4. Fields Institute Shared Graduate Courses (If you would like to offer a Fields Institute Shared Graduate Course, please select "Yes" and go to section 3). *

Yes

No

Fields Institute Shared Graduate Courses Program

See this link for info: https://www.fields.utoronto.ca/generalinfo/Shared-Graduate-Courses-Program

5. Detailed course outline for Fields Shared Courses program (Please either copy and paste your detailed course outline here or upload a file below.)

Enter your answer

6. Fields Shared Courses proposal (you can either upload a file here or submit your detailed course outline above.) (Non-anonymous question ⓘ)

B. Topics courses

Graduate faculty members are invited to offer special courses based on their research interests. Such courses should be accessible to certain groups of students who have no prerequisites other than the core courses.

7. Course level

Intermediate (**go to section 5**)

Introductory

Advanced

○ Special topics

8. Course(s) Title(s)

12 Definitions of the Alexander Polynomial.

9. Course(s) Number(s) (Please see the following link for a list of available numbers: https://sgs.calendar.utoronto.ca/degree/Mathematics). Please let us know if a new course code is needed.

MAT 1350

10. Subject Area

○ Algebra & Number Theory

○ Analysis & PDE

○ Computational & Applications

○ Operator Theory, Set Theory & Logic

○ Probability, Dynamics & Ergodic Theory

● Topology & Geometry

○ Other

11. Semester

- ○ Fall 2025
- ○ Winter 2026
- ◉ Either

12. Can this course be cross-listed with a 4th year UG course?

- ○ Yes
- ◉ No
- ○ Maybe
- ○ N/A

13. Can this course be offered as a seminar (CR/NCR)?

- ◉ Yes
- ○ No
- ○ Maybe

14. Detailed course outline (Please either copy and paste your detailed course outline - **including "prerequisites" and textbooks/references"** if applicable - here or upload a file below.)

In math, we often care for things for how well-connected they are. In themselves, 57 and 1729 and 196884 are just members of an infinitely long dull and monotone procession of "numbers". Yet the Theory of Numbers talks to nearly everything in mathematics, and everything talks to it. Knots are likewise dull, and Knot Theory is likewise interesting, and within Knot Theory the Alexander Polynomial plays a special role: it is arguably the most successful "Knot Invariant", it talks to everything, and everything talks to it.

In this class we will cover 12 of the 50 or so definitions of the Alexander Polynomial and discuss how they are related to formal probability and determinants, fundamental groups, and Fox derivatives, homology, covering spaces and finitely presented modules, Seifert surfaces and linking forms, braids and their Burau representation, exterior algebras and the Berezin integral, skein relations, tangles and meta-monoids, finite type invariants and the Kontsevich integral, 2-knots in 4D and the w-expansion and Hopf algebras and algebraic knot theory.

We will aim to implement in Mathematica almost everything that we will talk about, so this class is also about turning sophisticated mathematics into concise and effective code.

Prerequisites: Excellent grasp of everything in Core Algebra I (MAT1100) and in Core Topology (MAT1301), and no fear of computers.

Evaluation: Near-weekly problem sets, a possible full day of student lectures at the end.

15. Detailed course outline (**including "prerequisites" and textbooks/references" if applicable**) (Non-anonymous question ⓘ)

C. Intermediate courses

See options below.

16. Intermediate Course Title

Enter your answer

17. Intermediate Course(s) Number(s) (Please see the following link for a list of available numbers: https://sgs.calendar.utoronto.ca/degree/Mathematics). **Please let us know if a new course code is needed**.

Enter your answer

18. **Intermediate Course - ALGEBRA**

- ◯ Algebraic geometry (possibly 2 semesters, mentioned in the email announcement)
- ◯ Cohomology for Sheaves and Schemes (depending on how far the algebraic geometry course goes)
- ◯ Introduction to algebraic groups (also mentioned in geometry/topology)
- ◯ Class Field Theory
- ◯ Homological algebra
- ◯ Representation Theory and/or Lie theory (also mentioned in geometry/topology)
- ◯ Commutative Algebra

19. **Intermediate Course - ANALYSIS**

- ◯ Harmonic Analysis (mentioned in the email announcement, classical and/or application oriented)
- ◯ Functional analysis (maybe including Fredholm and index theory)
- ◯ Optimal Transportation and its applications (also listed in applied mathematics)
- ◯ Riemann surfaces
- ◯ Higher dimension phenomena in complex analysis (Kahler manifolds, Line bundles/divisors, metrics, curvature, Hormander's solvability of the \dbar equation adn consequences, higher dimension versions of the uniformization theorem, including Yau's solution of the Calabi conjecture)
- ◯ Several complex variables
- ◯ Hausdorff and Wiener measures
- ◯ Geometric Measure Theory (also listed in applied mathematics and in PDE)

- Other

20. **Intermediate Course - APPLIED MATHEMATICS**

- Methods of applied mathematics (MAT1800H)
- Applied analysis (Fourier transform, spectral theory, fixed point and implicit function theorems, bifurcation theory, variational calculus.)
- Optimal Transportation and its applications (also listed in analysis)
- Calculus of Variations and Geometric Measure Theory (also listed in analysis and in PDE)
- Numerical Methods (finite differences, finite elements, ...)
- Modeling
- Other

21. **Intermediate Course - DYNAMICAL SYSTEMS**

- Hyperbolic dynamical systems
- Ergodic theory (MAT1845H)
- Other

22. **Intermediate Course - GEOMETRY/TOPOLOGY**

- Symplectic manifolds
- Riemannian manifolds
- Second course in geometry ((i) Advanced topics in Riemannian geometry, (ii) introduction to symplectic geometry, (iii) vector bundles, connections, characteristic classes, Yang-Mills theory etc. (iv) Introduction to minimal surfaces, mean curvature flow, min-max etc.)

- Lie groups
- Complex manifolds / complex geometry
- Differential Topology (as opposed to introduction to manifolds, if the core course focuses on introduction to manifolds)
- Integrable systems
- Introduction to algebraic groups (also mentioned in algebra)
- Symmetric and/or Hermitian spaces
- Analysis and Geometry of Metric Spaces
- General Relativity
- Algebraic topology II (homotopy theory, cohomology theory, characteristic classes, spectral sequences, etc.)
- Complex Geometry (e.g. Griffith-Harris or Huybrechts)
- Lie groups and Lie algebras (also mentioned in algebra)
- Other

23. **Intermediate Course - PDEs**

- Nonlinear Dispersive and/or hyperbolic PDEs
- Inverse Scattering
- Fluid dynamics
- Euler and Navier-Stokes equations
- Elliptic PDE/ parabolic PDE
- Calculus of Variations and Geometric Measure Theory (also listed in analysis and in applied mathematics)

- Intro to variational methods (perhaps with a view towards optimal transport)
- Applied PDEs (PDEs from physical and life sciences and engineering: static solutions and solitons -including topological solitons- and their stability and dynamics, emerging solutions and pattern formation)
- Other

24. **Intermediate Course - PROBABILITY**

- Random matrices
- Stochastic differential equations/stochastic calculus
- Gaussian multiplicative chaos
- Introduction to models of statistical mechanics (percolation, SLE, spin glasses, etc.)
- Other

25. **Intermediate Course - OTHER AREAS**

- Combinatorics/ discrete mathematics
- Algebraic number theory II (local theory/class field theory/Sieve Theory/Modular forms)
- Higher Category Theory
- Other

26. Intermediate Course Description

Please enter your description below or choose the file upload option in the next section

Enter your answer

27. Intermediate course description (file upload) (Non-anonymous question ⓘ)

28. Other graduate courses that I am available to teach (Please list them in order of priority):

Enter your answer

29. **General Comments**

Enter your answer